Energy localization in disordered nonlinear lattices

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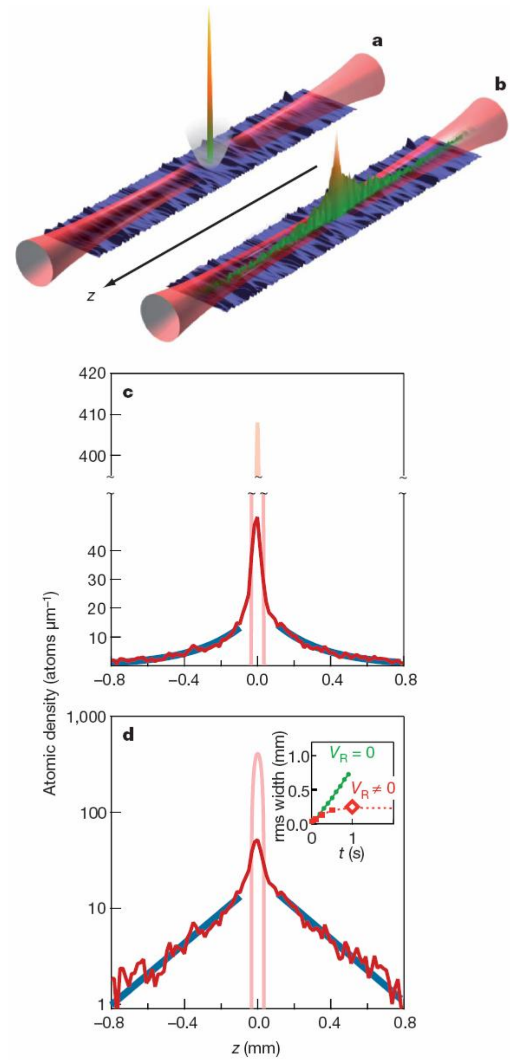
Interplay of disorder and nonlinearity

Waves in disordered media – Anderson localization

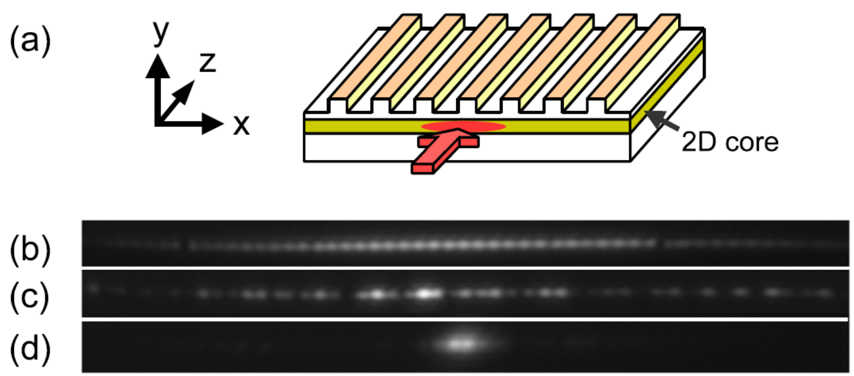
(Anderson Phys. Rev. 1958). Experiments on BEC (Billy et al. Nature 2008)



Waves in nonlinear disordered media – localization or delocalization?

Theoretical and/or numerical studies (Shepelyansky PRL 1993, Molina Phys. Rev. B 1998, Pikovsky & Shepelyansky PRL 2008, Kopidakis et al. PRL 2008)

Experiments: propagation of light in disordered 1d waveguide lattices (Lahini et al. PRL 2008)



The Klein – Gordon (KG) model

$$H_K = \sum_{l=1}^{N} \frac{p_l^2}{2} + \frac{\tilde{\varepsilon}_l}{2} u_l^2 + \frac{1}{4} u_l^4 + \frac{1}{2W} \left(u_{l+1} - u_l\right)^2$$

with **fixed boundary conditions** $u_0 = p_0 = u_{N+1} = p_{N+1} = 0$. Typically N=1000.

Parameters: **W** and the **total energy E.** $\tilde{\varepsilon}_l$ **chosen uniformly from** $\left[\frac{1}{2}, \frac{3}{2}\right]$.

The discrete nonlinear Schrödinger (DNLS) equation

$$H_D = \sum_{l=1}^{N} \varepsilon_l \left|\psi_l\right|^2 + \frac{\beta}{2} \left|\psi_l\right|^4 - \left(\psi_{l+1}\psi_l^* + \psi_{l+1}^*\psi_l\right)^2$$

where ε_l **chosen uniformly from** $\left[-\frac{W}{2}, \frac{W}{2}\right]$ and β **is the nonlinear parameter.**

Scales

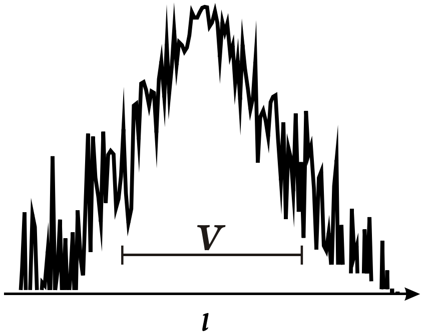
Linear case: $\omega_\nu^2 \in \left[\frac{1}{2}, \frac{3}{2} + \frac{4}{W}\right]$, **width of the squared frequency spectrum:**

$$\Delta_K = 1 + \frac{4}{W}$$

$$(\Delta_D = W + 4)$$

Localization volume of an eigenstate:

$$V \sim \frac{1}{\sum_{l=1}^{N} A_{\nu,l}^4}$$



Average spacing of squared eigenfrequencies of NMs within the range of a localization volume: $d_K \approx \frac{\Delta_K}{V}$

Nonlinearity induced squared frequency shift of a single site oscillator

$$\delta_l = \frac{3E_l}{2\tilde{\varepsilon}_l} \propto E \qquad (\delta_l = \beta |\psi_l|^2)$$

The relation of the two scales $d_K \leq \Delta_K$ with the nonlinear frequency shift δ_l determines the packet evolution.

Distribution characterization

We consider normalized energy distributions in normal mode (NM) space $z_\nu \equiv \frac{E_\nu}{\sum_m E_m}$ with $E_\nu = \frac{1}{2}\left(\dot{A}_\nu^2 + \omega_\nu^2 A_\nu^2\right)$, where A_ν is the amplitude of the νth NM.

Second moment: $m_2 = \sum_{\nu=1}^{N} (\nu - \bar{\nu})^2 z_\nu$ with $\bar{\nu} = \sum_{\nu=1}^{N} \nu z_\nu$

Participation number: $P = \frac{1}{\sum_{\nu=1}^{N} z_\nu^2}$

measures the number of stronger excited modes in z_ν. Single mode $P=1$, Equipartition of energy $P=N$.

3 Different Dynamical Regimes

Weak Chaos Regime: $\delta<d$, $m_2 \sim t^{1/3}$

Frequency shift is less than the average spacing of interacting modes. NMs are weakly interacting with each other. [Molina PRB (1998) – Pikovsky, Shepelyansky, PRL (2008)].

Intermediate Strong Chaos Regime: $d<\delta<\Delta$, $m_2 \sim t^{1/2} \longrightarrow m_2 \sim t^{1/3}$

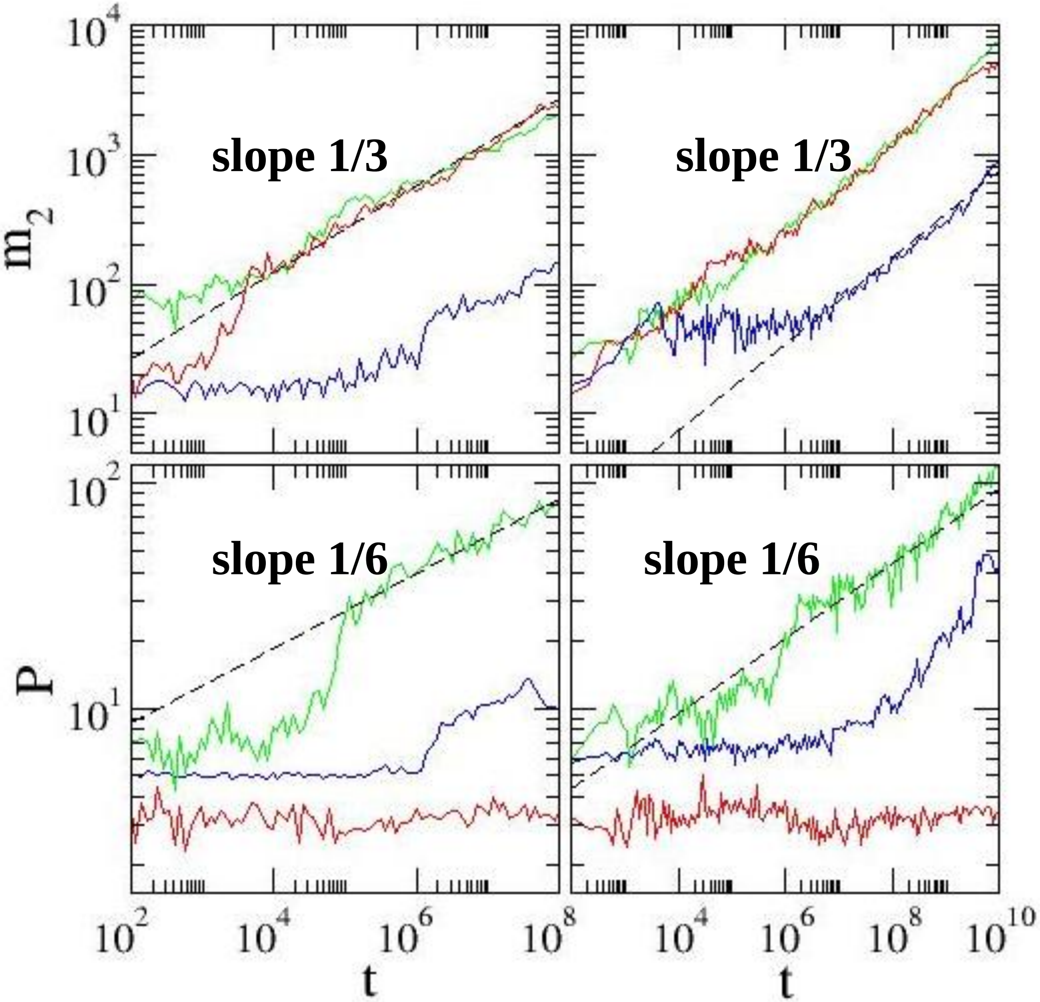
Almost all NMs in the packet are resonantly interacting. Wave packets initially spread faster and eventually enter the weak chaos regime.

Selftrapping Regime: $\delta>\Delta$

Frequency shift exceeds the spectrum width. Frequencies of excited NMs are tuned out of resonances with the nonexcited ones, leading to selftrapping, while a small part of the wave packet subdiffuses [Kopidakis et al., PRL (2008)].

Single site excitations

DNLS W=4, β= 0.1, 1, 4.5   KG W = 4, E = 0.05, 0.4, 1.5



No strong chaos regime

In weak chaos regime we averaged the measured exponent α ($m_2 \sim t^{\alpha}$) over 20 realizations:

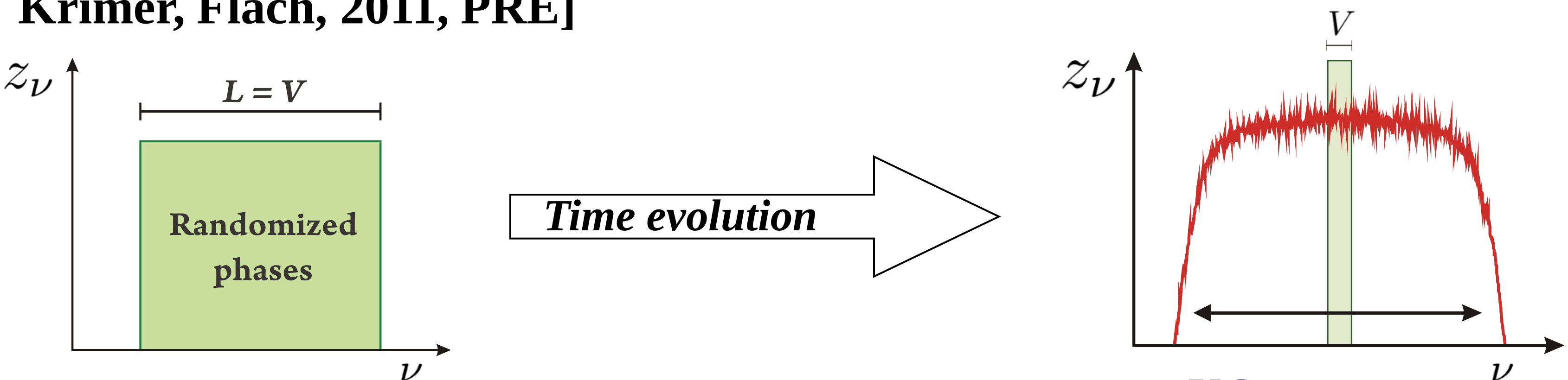
α=0.33±0.05 (KG)

α=0.33±0.02 (DLNS)

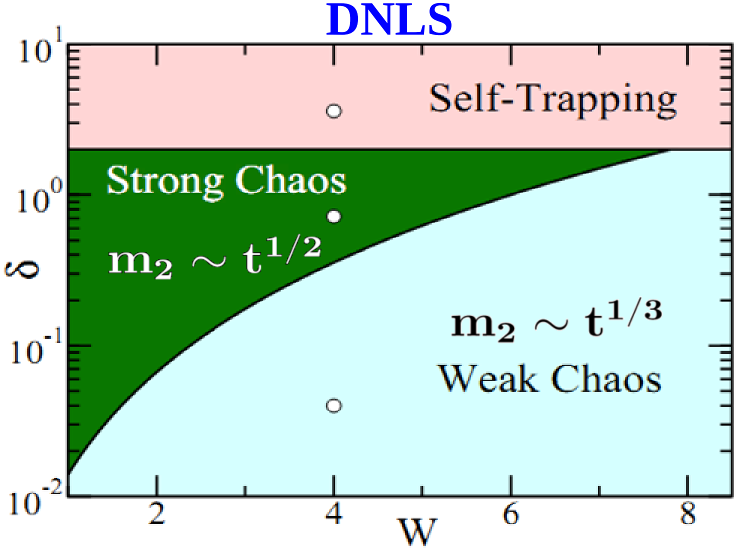
[Flach, Krimer, Ch. S., 2009, PRL – Ch. S., Krimer, Komineas, Flach, 2009, PRE – Ch. S., Flach, 2010, PRE]

Crossover from strong to weak chaos

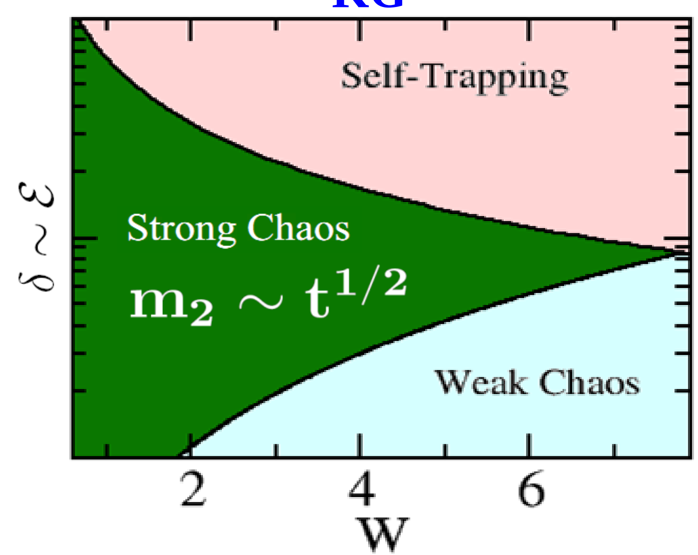
We consider compact initial wave packets of width *L=V* [Laptyeva, Bodyfelt, Krimer, Ch. S., Flach, 2010, EPL – Bodyfelt, Laptyeva, Ch. S., Krimer, Flach, 2011, PRE]



DNLS

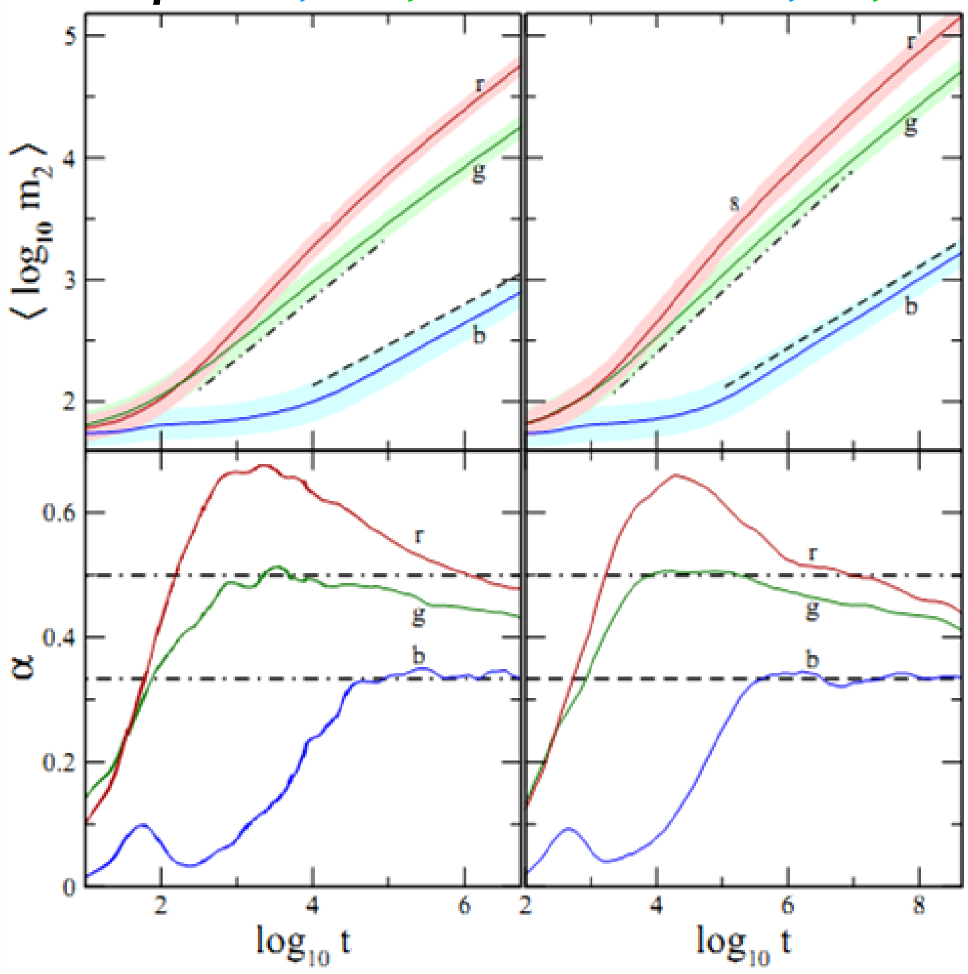


KG



Crossover from strong to weak chaos

DNLS β= 0.04, 0.72, 3.6 **KG E= 0.01, 0.2, 0.75**



W=4

Average over 1000 realizations!

$$\alpha(\log t) = \frac{d \langle \log m_2 \rangle}{d \log t}$$

α=1/2

α=1/3

Conclusions

- We predicted theoretically and verified numerically the existence of three different dynamical behaviors:
  - ✓ Weak Chaos Regime: $\delta < d$, $m_2 \sim t^{1/3}$
  - ✓ Intermediate Strong Chaos Regime: $d < \delta < \Delta$, $m_2 \sim t^{1/2} \longrightarrow m_2 \sim t^{1/3}$
  - ✓ Selftrapping Regime: $\delta > \Delta$
- Generality of results: a) Two different models: KD and DNLS, b) Predictions made for DNLS are verified for both models.
- Our results suggest that Anderson localization is eventually destroyed by the slightest amount of nonlinearity, since spreading does not show any sign of slowing down.

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